Point-Coupling Models from Mesonic Hypermassive Limit and Mean-Field Approaches

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Abstract

In this work we show how nonlinear point-coupling models, described by a Lagrangian density that presents only terms up to fourth order in the fermion condensate ($\bar{\psi}\psi$), are derived from a modified meson-exchange nonlinear Walecka model. The derivation can be done through two distinct methods, namely, the hypermassive meson limit within a functional integral approach, and the mean-field approximation in which equations of state at zero temperature of the nonlinear point-coupling models are directly obtained.

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I. INTRODUCTION

The point-coupling interaction problem was first addressed in the early thirties by L. H. Thomas [1] investigating the range of the two-nucleon force. As a side remark of his work, he observed that when the range of the two-body force goes to zero with the two-body binding energy kept fixed the binding energy of the quantum-three-body state goes to minus infinity. Decades later, a new and apparently not related three-body effect was proposed by Efimov [2]. When a quantum two-body system has a zero energy bound state then the three-body system will have an infinite number of bound states with an accumulation point at the common two and three body threshold. Both, the Efimov and the Thomas effects are universal, since the associated three-body wave functions have long tails in the classically forbidden region outside the range of the potential. In a unified momentum space description, based on ideas of Amado and Noble [3], it has been claimed that these two apparently different effects are related to the same singular structure of the kernel of the Faddeev equation [4]. On the other hand, in appropriate units, the presence of one of these effects implies the presence of the other [4, 5]. The Thomas-Efimov effect explains very well some few-body correlations [6] and is conjectured to be behind the Coester band [7] for different nuclear matter models [8].

Since relativistic hadronic point-coupling models, that have been used in the description of infinite nuclear matter, and finite nuclei as well [9], can be viewed as a connection between the well established finite range relativistic models and the Skyrme ones [9], a better understanding of their structure becomes of interest when an important theoretical challenge is to construct a universal nuclear effective density functional [10].

In this work we deal with a specific nonlinear point-coupling model (NLPC) described by a Fermionic Lagrangian density with interaction terms in third and fourth powers of the scalar density operator, that we have used in Ref. [11] in a comparative study with the standard nonlinear Walecka model(s), and in Ref. [12], in which after taking its nonrelativistic limit, a generalized Skyrme energy density functional was obtained.

We focus on the derivation of the NLPC model from finite range ones in two different ways. First, we present the infinitely massive meson limit within the formal point of view of the integral functional approach. This method shows in a clear fashion the equivalence of the usual Walecka model to the linear point-coupling one. However, contrary to this case,

NLW models are thereby not formally equivalent to NLPC ones. Therefore, we pose the question on how NLPC models can be derived if one insists in a meson exchange model as their origin. To answer this question, we construct a modified nonlinear Walecka (MNLW) model in which the limit of infinite meson masses leads exactly to the Lagrangian density of the NLPC model. This MNLW model includes terms of third and fourth powers of the scalar meson field, together with terms in lower powers of the fields coupled to the Fermionic scalar density operator. Preliminary results on the use of this hypermassive meson limit have been presented in Ref. [13].

The traditional mean-field approximation, implemented with some physical requirements, is an alternative way to construct the NLPC models from the MNLW ones. In this case we show that the equations of state of the MNLW models, relating energy, density and pressure, are exactly the same of the NLPC ones.

We can thus synthesize the study of the NLW, MNLW, and NLPC models through the following diagram

$$
\begin{tikzcd}
\boxed{NLW} \arrow[r, "+\ other\ terms"] \arrow[dr, dashed, leftrightarrow, "numerical"'] & \boxed{MNLW} \arrow[d, leftrightarrow, "hypermassive\ limit"', "mean-field"] \\
& \boxed{NLPC}
\end{tikzcd}
$$

where the numerical equivalences between NLW and NLPC models have been analyzed in Ref. [11], and the different connections among MNLW and NLPC models will be studied in this work at an analytical level.

This work extends the study presented in [11, 13] through the following points:

• A detailed and rigorous derivation of the point-coupling models from the modified NLW ones is done by using the hypermassive meson limit in the functional integral method. From this approach we show how the linear PC models can be obtained from the Walecka one(s), and in the same way, how the MNLW model(s) generates the NLPC one(s).

• The mean-field approach in the no-sea approximation is also used to construct the equations of state (EOS) of infinite nuclear matter of the MNLW model. In this approximation we show that these EOS are exactly the same of the NLPC ones.

Our paper is organized as follows. In Sec. II by using a functional integral formalism we

derive the linear point-coupling model from the Walecka one. The same study is extended to obtain the NLPC models from the MNLW ones. In Sec. III we explicitly derive the equations of state of the MNLW model. Finally, the main conclusions are summarized.

II. HYPERMASSIVE MESON LIMIT

A. Linear point-coupling model from the Walecka one

We start with the Walecka model Lagrangian density given by [14]

$$\begin{aligned}\mathcal{L}_W &= \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_s^2\phi^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu - g_s\bar{\psi}\phi\psi \\ &- g_V\bar{\psi}\gamma^\mu V_\mu\psi \end{aligned} \qquad (1)$$

where $F^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$.

In this Lagrangian, ψ, ϕ and V^μ are the nucleon, scalar and vector fields respectively and M, m_s and m_v refer to bare nucleon and the mesonic σ and ω masses respectively.

The fermionic, scalar and vector fields constitute an irreducible set of generators $\{\psi, \bar{\psi}, \phi, V_\mu\}$ of the intrinsic local algebra of fields of the model, $\mathbf{A}$. The polynomial algebra of intrinsic fields allows for the construction of the net of Wightman functions. From the Wightman functions of the polynomial algebra of intrinsic fields the physical Hilbert space is reconstructed, $\mathbf{H} = \mathbf{A}\,|0\rangle$, thus defining the physical content of the model [15]. Under this setting an equivalence of models should be understood as an assertion on the isomorphism of their physical Hilbert spaces. This kind of equivalence is extremely rare to occur. It is believed to occur in the context of duality transformation and is witnessed in the context of two-dimensional field theory as in the bosonization phenomenon [16]. In establishing this isomorphism, operator and functional integral methods are usually complementary tools [17].

Our concern here, however, is with much less stringent equivalences. We shall, first, derive an equivalence of the usual Walecka model to the linear point-coupling model with terms of second order in the fermionic density and vector current (fourth order in the fermion fields). This is not an equivalence of the Hilbert spaces but of the physical content of the models amenable to mean-field procedures. The mean-field procedures start from discarding the kinetic terms for the scalar and vector fields what is related to the infinite limit of

the masses of the bosons. The derivation of this equivalence will be provided here using functional integral methods. The presentation of this treatment at length in this section for this case is motivated by the need of clarifying the ideas to be used later in the case of interaction Lagrangian models, which contains higher power of the mesonic fields. It will be thus distinguished what is and what is not valid for these more involved models.

We start by constructing the generating functionals within the functional integral formalism from which the correlation functions are obtained. We will then use this formalism to connect the Walecka model to the linear point-coupling model.

For the Walecka model, the generating functional is given by

$$W[J, A_\mu, \eta, \bar{\eta}] = N \int [D\psi][D\bar{\psi}][DV^\mu][D\phi]\ \mathrm{e}^{iS_S} \tag{2}$$

with

$$N^{-1} = \int d^4x\ \mathrm{e}^{iS}\,, \tag{3}$$

$$S = \int d^4x\ \mathcal{L}_W \quad \text{and} \tag{4}$$

$$S_S = \int d^4x\ \left[\mathcal{L}_W + A_\mu(x)V^\mu(x) + J(x)\phi(x) + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right] \tag{5}$$

where $A_\mu(x)$, $J(x)$, $\eta(x)$ e $\bar{\eta}(x)$ are the sources for vectorial (V^μ), scalar (ϕ) and spinorial ($\bar{\psi}$ and ψ) fields respectively. S_S and S are the actions with and without the source terms. Here it will be important to make the definitions $V'^\mu \equiv m_V V^\mu$, $\phi' \equiv m_s\phi$, $G'_s \equiv g_s/m_s$, $G'_V \equiv g_V/m_V$, $A'_\mu \equiv A_\mu/m_V$ and $J' \equiv J/m_s$. With these definitions we can write

$$\mathcal{L}_W = \mathcal{L}'_W + \frac{1}{2m_s^2}\partial^\mu\phi'\partial_\mu\phi' - \frac{1}{4m_V^2}F'^{\mu\nu}F'_{\mu\nu} \equiv \mathcal{L}'_W + U(\phi', V'^\mu) \tag{6}$$

where

$$F'^{\mu\nu} = \partial^\mu V'^\nu - \partial^\nu V'^\mu \quad \text{and} \tag{7}$$

$$\mathcal{L}'_W = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi - \frac{1}{2}\phi'^2 + \frac{1}{2}V'_\mu V'^\mu - G'_s\bar{\psi}\phi'\psi - G'_V\bar{\psi}\gamma^\mu V'_\mu\psi\,. \tag{8}$$

Now, the generating functional Eq. (2) may be written in the following form,

$$W[J', A'_\mu, \eta, \bar{\eta}] = N \int [D\psi][D\bar{\psi}][DV'^\mu][D\phi']\ \mathrm{e}^{\,i\left[\int d^4x\ U(\phi', V'^\mu) + S'_S\right]} \tag{9}$$

where

$$S'_S = \int d^4x\ \left[\mathcal{L}'_W + A'_\mu(x)V'^\mu(x) + J'(x)\phi'(x) + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right]\ . \tag{10}$$

In order to make contact with mean-field methods we consider the limit in which the mesonic masses become very large, allowing that terms involving $1/m_s^2$ and $1/m_V^2$ may be treated perturbatively in a generating functional expansion. It is in this perspective that we identify the fields in $U(\phi', V'^\mu)$ with the respective functional derivatives,

$$
\begin{aligned}
U(\phi', V'^\mu) &= \frac{1}{2m_s^2}\partial^\mu\phi'\partial_\mu\phi' - \frac{1}{4m_V^2}F'^{\mu\nu}F'_{\mu\nu} \\
&= \frac{1}{2m_s^2}\left(\partial^\mu\frac{\delta}{\delta J'}\right)^2 - \frac{1}{4m_V^2}\left(\partial^\nu\frac{\delta}{\delta A'_\mu} - \partial^\mu\frac{\delta}{\delta A'_\nu}\right)^2 \\
&= U\left(\frac{\delta}{\delta J'}, \frac{\delta}{\delta A'_\mu}\right),
\end{aligned} \tag{11}
$$

which allows us to write (9) as

$$
W[J', A'_\mu, \eta, \bar{\eta}] = N\ \mathrm{e}^{i\left[\int d^4x\ U\left(\frac{\delta}{\delta J'}, \frac{\delta}{\delta A'_\mu}\right)\right]} \int [D\psi][D\bar{\psi}][DV'^\mu][D\phi']\ \mathrm{e}^{iS'_S}. \tag{12}
$$

Up to now we have not changed the physical content of the Walecka model but merely rewritten the generating functional in a form suited to a non standard expansion. We consider in the following the zero order term of the expansion of the generating functional by using

$$
\mathrm{e}^{i\left[\int d^4x\ U\left(\frac{\delta}{\delta J'}, \frac{\delta}{\delta A'_\mu}\right)\right]} \simeq 1, \tag{13}
$$

so that

$$
W_{MF}[J', A'_\mu, \eta, \bar{\eta}] = N \int [D\psi][D\bar{\psi}][DV'^\mu][D\phi']\ \mathrm{e}^{iS'_S} \tag{14}
$$

in which the kinetic terms associated to the mesonic fields are neglected and W_{MF} refers to the generating functional associated to mean-field treatment.

It is important to analyze the situation once again from the view point of the structural properties of the model. In the start the intrinsic algebra of fields was generated by the irreducible set of fields $\mathbf{S_1} = \{\psi, \bar{\psi}, \phi, V^\mu\}$. Now, in the zero order approximation, since the mesonic kinetic terms have been suppressed, the equations of motion will allow to express, in this case explicitly, the mesonic fields in terms of the fermion densities. The algebra of fields has been turned on a reducible algebra and the irreducible algebra of fields is now constructed from the fermionic fields only $\mathbf{S_2} = \{\bar{\psi}, \psi\}$. This seems to be a mathematical counterpart of the spirit of mean-field treatment. Of course the mesonic fields are still in play in the dynamics of the model, since they are still coupled to the fermionic fields, but

we have lost control on the independent degrees of freedom associated to the mesonic fields. The physical picture associated to this mathematical aspect will be discussed at the end of this and the next subsections.

We proceed to the last step of the process, expressing the mesonic fields in terms of the fermion degrees and expressing the dynamics solely in terms of fermion fields. We do this in the functional integral formalism by decoupling the mesonic fields reducing them to auxiliary fields devoid of physical content. To decouple them, we will proceed with a transformation of the mesonic fields to the auxiliary ones. Before this, since we are not interested in analyzing the mesonic correlation functions we will do $J'(x) = A'_\mu(x) = 0$ in Eq. (10). With this condition and the following identities,

$$-\frac{1}{2}\phi'^2 - G'_s\bar{\psi}\phi'\psi \;=\; -\frac{1}{2}(\phi' + G'_s\bar{\psi}\psi)^2 + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 \tag{15}$$

$$\frac{1}{2}V'^\mu V'_\mu - G'_V\bar{\psi}\gamma^\mu V'_\mu\psi \;=\; \frac{1}{2}(V'^\mu - G'_V\bar{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 \;, \tag{16}$$

Eq. (10) may be rewritten as

$$\begin{aligned} S'_S \;=\; & \int d^4x \; \Big[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi - \frac{1}{2}(\phi' + G'_s\bar{\psi}\psi)^2 + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 \\ & + \; \frac{1}{2}(V'^\mu - G'_V\bar{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\Big] \,. \end{aligned} \tag{17}$$

Now, we define the change from the variables ϕ' and V'^μ to λ and R^μ

$$\lambda \;=\; \phi' + G'_s\bar{\psi}\psi \tag{18}$$

$$R^\mu \;=\; V'^\mu - G'_V\bar{\psi}\gamma^\mu\psi \,. \tag{19}$$

With this the generating functional, Eq. (14), becomes

$$W[\eta,\bar{\eta}] \;=\; N\int[D\lambda]\;\mathrm{e}^{-i\int d^4x\,\frac{\lambda^2}{2}}\int[DR^\mu]\;\mathrm{e}^{i\int d^4x\frac{1}{2}R^\mu R_\mu}\int[D\psi][D\bar{\psi}]\;\mathrm{e}^{iS''_S}, \tag{20}$$

$$\;=\; \mathcal{N}\int[D\psi][D\bar{\psi}]\;\mathrm{e}^{iS''_S}, \tag{21}$$

where

$$\mathcal{N}^{-1} \;=\; \int[D\psi][D\bar{\psi}]\mathrm{e}^{iS''} \tag{22}$$

$$S'' \;=\; \int d^4x\;\Big[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2\Big] \quad \text{and} \tag{23}$$

$$S''_S \;=\; \int d^4x\;\Big[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 + \bar{\eta}\psi + \eta\bar{\psi}\Big] \;. \tag{24}$$

Now, the Lagrangian density associated to Eqs. (23)-(24) is given simply by

$$\mathcal{L}_{PC} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2, \tag{25}$$

presenting no mesonic fields.

Let us remark here that the decoupling procedure is just the integration off of the mesonic fields leading to contributions in the Lagrangian density, Eq. (25), that are quadratic in the fermion density and fermion vector current. Indeed Eq. (20) expresses the auxiliary character of the mesonic fields. Their contribution, in the zero order approximation, is resumed in the quadratic terms, and no true quanta can be assigned to the mesons in this approximation. We have obtained thus a rigorous derivation of the equivalence between the linear point-coupling model and the zero-order approximation to the Walecka model. This means an equivalence between the fermion correlation functions, and Wightman functions, of the models. A reconstruction of the Hilbert space of the zero-order Walecka and linear point-coupling models will lead to an isomorphism of their Hilbert spaces. On the other hand, this implies a rigorous derivation of the equivalence between the mean-field Walecka model and the linear point-coupling one. It means that both models will lead to the same equations of state as has been pointed out in [18]. From the physical point of view it should be said that the meson degrees of freedom are not excited in the infinite mass limit. This is the physical mechanism that leads to the isomorphism of the Hilbert spaces of the models that were pointed here.

B. NLPC model from the MNLW ones

In the previous section we have shown that the point-coupling model was obtained from the Walecka one in the limit of hypermassive mesons. To improve the experimental finite nuclei [19] and infinite nuclear matter bulk properties, the well known nonlinear Walecka model [20] adds cubic and quartic scalar self-coupling to the Walecka model,

$$\mathcal{L}_{NLW} = \mathcal{L}_W - \frac{A}{3}\phi^3 - \frac{B}{4}\phi^4. \tag{26}$$

Indeed, there is a family of acceptable NLW models which differ in respect to how the A and B free parameters are chosen to fit different experimental nuclear data [21]. As we have pointed out before, higher order point-coupling models involving $(\bar{\psi}\psi)^3$ and $(\bar{\psi}\psi)^4$ (NLPC) have been also successfully applied to finite nuclei [9].

Still at the finite range level, i.e., finite meson masses, different kinds of Walecka-type models such as variants of NLW ones [22, 23], with density dependent coupling constants [24], and the linear chiral model [25], were also used in the description of nuclear frameworks. The NLW models derived from a quark model perspective can be found in Ref. [26]. Particularly, the authors show that the Walecka model is the limit of infinite quark mass, where the quarkdynamics freezes.

The question we pose in this section is whether a NLW model, Eq. (26), leads to a nonlinear point-coupling model in the limit of infinite meson masses, which includes cubic and quartic self-fermionic terms,

$$\mathcal{L}_{NLPC} = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 + \frac{A'}{3}(\bar{\psi}\psi)^3 + \frac{B'}{4}(\bar{\psi}\psi)^4. \quad (27)$$

The answer is not. Indeed reproducing the procedure of the last section instead of equation (14) with (17) we obtain, integrating away the vectorial field

$$W_{MF-NL}[\eta,\bar{\eta}] = N\int[D\psi][D\bar{\psi}][D\phi']\ \mathrm{e}^{iS'_{S-NL}} \quad (28)$$

with

$$\begin{aligned} S'_{S-NL} &= \int d^4x\ \Big[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi - \frac{1}{2}(\phi' + G'_s\bar{\psi}\psi)^2 + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 \\ &+ \ -\frac{A}{3}\phi'^3 - \frac{B}{4}\phi'^4 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\Big]. \end{aligned} \quad (29)$$

Now the functional integral for the field ϕ' can not be explicitly performed and all we can say is that it gives rises to an unknown functional of $\bar{\psi}\psi$. With this the identification between NLW and NLPC fails. That is we can not assert the formal equivalence between NLW and NLPC even at the restricted sense of zero-order expansion in the kinetic terms. An approach, that connects the NLW and NLPC models, has been nicely performed in Ref. [27] where the authors use an expansion in the meson propagators treating the nonlinearity in the ϕ field by an iterative process.

In order to gain a deeper understanding on how to obtain the NLPC model, Eq. (27), in the meson hypermassive limit, we consider here a modification of $\mathcal{L}_{NLW}$ that includes second and third powers of the scalar meson field coupled to the appropriate powers of the Fermion scalar density, which allows to decouple the scalar meson field, when the scalar mass goes to infinity.

We consider then the modified nonlinear Lagrangian,

$$\mathcal{L}_{MNLW} \;=\; \mathcal{L}_W + \mathcal{L}_3 + \mathcal{L}_4 \tag{30}$$

where

$$\mathcal{L}_3 \;=\; -\frac{A'}{3}\left[\left(\frac{m_s^2}{g_s}\right)^3 \phi^3 + 3\left(\frac{m_s^2}{g_s}\right)^2 \phi^2\bar{\psi}\psi + 3\frac{m_s^2}{g_s}\phi(\bar{\psi}\psi)^2\right] \quad \text{and} \tag{31}$$

$$\mathcal{L}_4 \;=\; -\frac{B'}{4}\left[\left(\frac{m_s^2}{g_s}\right)^4 \phi^4 + 4\left(\frac{m_s^2}{g_s}\right)^3 \phi^3\bar{\psi}\psi + 6\left(\frac{m_s^2}{g_s}\right)^2 \phi^2(\bar{\psi}\psi)^2 + 4\frac{m_s^2}{g_s}\phi(\bar{\psi}\psi)^3\right] \;. \tag{32}$$

The generating functional for this Lagrangian is the same given by Eq. (2) with $\mathcal{L}_W$ substituted by $\mathcal{L}_{MNLW}$.

By using again the definitions for V'^{μ}, ϕ', G'_s, G'_V, A'_μ and J', we see that Eq. (30) becomes

$$\begin{aligned}\mathcal{L}_{MNLW} \;&=\; \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2m_s^2}\partial^\mu\phi'\partial_\mu\phi' - \frac{1}{2}\phi'^2 - \frac{1}{4m_V^2}F'^{\mu\nu}F'_{\mu\nu} + \frac{1}{2}V'_\mu V'^\mu - G'_s\bar{\psi}\phi'\psi \\ &-\; G'_V\bar{\psi}\gamma^\mu V'_\mu\psi + \mathcal{L}'_3 + \mathcal{L}'_4 \qquad (33)\\ &\equiv\; \mathcal{L}'_{MNLW} + \mathcal{L}'_3 + \mathcal{L}'_4 + U(\phi', V'^\mu) \qquad (34)\end{aligned}$$

where

$$\mathcal{L}'_{MNLW} \;=\; \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi - \frac{1}{2}\phi'^2 + \frac{1}{2}V'_\mu V'^\mu - G'_s\bar{\psi}\phi'\psi - G'_V\bar{\psi}\gamma^\mu V'_\mu\psi\,, \tag{35}$$

$$\mathcal{L}'_3 \;=\; -\frac{A'}{3}\left[\frac{1}{G_s'^3}\phi'^3 + \frac{3}{G_s'^2}\phi'^2\bar{\psi}\psi + \frac{3}{G'_s}\phi'(\bar{\psi}\psi)^2\right]\,, \tag{36}$$

$$\mathcal{L}'_4 \;=\; -\frac{B'}{4}\left[\frac{1}{G_s'^4}\phi'^4 + \frac{4}{G_s'^3}\phi'^3\bar{\psi}\psi + \frac{6}{G_s'^2}\phi'^2(\bar{\psi}\psi)^2 + \frac{4}{G'_s}\phi'(\bar{\psi}\psi)^3\right] \tag{37}$$

with $U(\phi', V'^\mu)$ given by Eq. (11). Therefore, the generating functional can be rewritten,

$$W[J', A'_\mu, \eta, \bar{\eta}] = N\int[D\psi][D\bar{\psi}][DV'^\mu][D\phi']\; \mathrm{e}^{\,i\left[\int d^4x\; U(\phi', V'^\mu) + S'_S\right]} \tag{38}$$

where

$$S'_S = \int d^4x\;\left[\mathcal{L}'_{MNLW} + \mathcal{L}'_3 + \mathcal{L}'_4 + A'_\mu(x)V'^\mu(x) + J'(x)\phi'(x) + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right]\;. \tag{39}$$

Again, $U(\phi', V'^\mu)$ will be treated perturbatively in the generating functional. The zero order approximation, Eq. (13), leads to

$$W[J', A'_\mu, \eta, \bar{\eta}] = N\int[D\psi][D\bar{\psi}][DV'^\mu][D\phi']\; \mathrm{e}^{iS'_S}\;. \tag{40}$$

As we have done previously, we will discard the control on the mesonic correlation functions by taking $J'(x) = A'_\mu(x) = 0$ in Eq. (39). Now, together with Eqs. (15)-(16) we will use the following set of identities,

$$-\frac{A'}{3}\left[\frac{1}{{G'_s}^3}\phi'^3 + \frac{3}{{G'_s}^2}\phi'^2\bar{\psi}\psi + \frac{3}{G'_s}\phi'(\bar{\psi}\psi)^2\right] = -\frac{A'}{3{G'_s}^3}(\phi' + G'_s\bar{\psi}\psi)^3 + \frac{A'}{3}(\bar{\psi}\psi)^3\,, \tag{41}$$

$$\begin{aligned} -\frac{B'}{4}\left[\frac{1}{{G'_s}^4}\phi'^4 + \frac{4}{{G'_s}^3}\phi'^3\bar{\psi}\psi + \frac{6}{{G'_s}^2}\phi'^2(\bar{\psi}\psi)^2 + \frac{4}{G'_s}\phi'(\bar{\psi}\psi)^3\right] \;&=\; -\frac{B'}{4{G'_s}^4}(\phi' + G'_s\bar{\psi}\psi)^4 \\ &+\; \frac{B'}{4}(\bar{\psi}\psi)^4, \end{aligned} \tag{42}$$

which allows us rewrite Eq. (39) as

$$\begin{aligned} S'_S \;=\; \int d^4x\; &\Big[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi - \frac{1}{2}(\phi' + G'_s\bar{\psi}\psi)^2 + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 \\ +\; &\frac{1}{2}(V'^\mu - G'_V\bar{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 - \frac{A'}{3{G'_s}^3}(\phi' + G'_s\bar{\psi}\psi)^3 + \frac{A'}{3}(\bar{\psi}\psi)^3 \\ -\; &\frac{B'}{4{G'_s}^4}(\phi' + G'_s\bar{\psi}\psi)^4 + \frac{B'}{4}(\bar{\psi}\psi)^4 + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\Big]\,. \end{aligned} \tag{43}$$

If we define once again the change of fields leading to the auxiliary fields,

$$\lambda \;=\; \phi' + G'_s\bar{\psi}\psi \tag{44}$$

$$R^\mu \;=\; V'^\mu - G'_V\bar{\psi}\gamma^\mu\psi, \tag{45}$$

we will have for the mesonic field integrals in Eq. (38) the following forms,

$$\begin{aligned} &\int [D\phi']\;\mathrm{e}^{-i\int d^4x\;\left[\frac{1}{2}(\phi' + G'_s\bar{\psi}\psi)^2 + \frac{A'}{3{G'_s}^3}(\phi' + G'_s\bar{\psi}\psi)^3 + \frac{B'}{4{G'_s}^4}(\phi' + G'_s\bar{\psi}\psi)^4\right]} = \\ &\qquad = \int [D\lambda]\;\mathrm{e}^{-i\int d^4x\;\left[\frac{1}{2}\lambda^2 + \frac{A'}{3{G'_s}^3}\lambda^3 + \frac{B'}{4{G'_s}^4}\lambda^4\right]} \end{aligned} \tag{46}$$

and

$$\int [DV'^\mu]\;\mathrm{e}^{i\int d^4x\;\frac{1}{2}(V'^\mu - G'_V\bar{\psi}\gamma^\mu\psi)^2} = \int [DR^\mu]\;\mathrm{e}^{i\int d^4x\;\frac{1}{2}R^\mu R_\mu}. \tag{47}$$

The identities and translations above allows to rewrite the Eq. (40) as

$$W[\eta,\bar{\eta}] = \mathcal{N}\int [D\psi][D\bar{\psi}]\;\mathrm{e}^{iS''_S} \tag{48}$$

where

$$\mathcal{N}^{-1} = \int [D\psi][D\bar{\psi}]\ \mathrm{e}^{iS''}\ , \tag{49}$$

$$S'' = \int d^4x\ \left[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 + \frac{A'}{3}(\bar{\psi}\psi)^3 + \frac{B'}{4}(\bar{\psi}\psi)^4\right] \tag{50}$$

$$S''_S = \int d^4x\ \left[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2(\bar{\psi}\psi)^2 - \frac{1}{2}{G'_V}^2(\bar{\psi}\gamma^\mu\psi)^2 + \frac{A'}{3}(\bar{\psi}\psi)^3 \right.$$
$$\left. + \frac{B'}{4}(\bar{\psi}\psi)^4 + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right]\ . \tag{51}$$

The Lagrangian density contained in Eqs. (50)-(51) describes the fermionic nonlinear point-coupling model we have aimed. We have seen that the generating Lagrangian to obtain the NLPC model through the mesonic hypermassive limit is $\mathcal{L}_{MNLW}$, given by Eq. (30) and not $\mathcal{L}_{NLW}$ as could be naively expected.

Let us perform here an analysis based upon the structural properties of the model. We are not asserting here an equivalence of NLPC and MNLW models. The Hilbert spaces of both models are not isomorphic. But the zero order expansion of the MNLW model has been exactly mapped onto the NLPC models. Once again from the view point of an structural analysis the irreducible algebra of fields of MNLW composed of the polynomial algebra of the fermion and meson fields becomes reducible in the zero order approximation. The mesonic fields turn out to be functions of the fermion fields and the irreducible algebra is composed solely of the fermion field algebra. In the language of functional integrals this is implemented by the decoupling of the auxiliary fields λ and R^μ. The equations of motion of the auxiliary fields bring about the functional relation between the original mesonic fields and the fermion bilinears. Contrary to the linear case treated in the preceding section now the equations of motion do not demand $\lambda = 0$ and $R^\mu = 0$. The equations for the auxiliary fields includes, in principle, other roots besides the trivial ones. Actually, as we will discuss in the next section, the equations of state of the MNLW model in the mean-field approximation, depend on the mean value of the auxiliary field λ and differ from those of the NLPC model only by the terms containing this field. However, the physical requirement of vanishing pressure at zero Fermi momentum is satisfied only by the trivial solution for the auxiliary field λ.

Another aspect that should be stressed is concerned to the renormalization properties of the models. The infinite mass expansion that is done here effectively changes the power counting dimensions of the mesonic fields. Since their kinetic terms are discarded, there appear no inverse powers of the momenta in their propagators in the ultraviolet region.

The result is that the $\mathcal{L}_{NLPC}$ models are non-renormalizable while the $\mathcal{L}_{NLW}$ are (power-counting) renormalizable. The physical reason for this change shall be emphasized: our approximation freezes the meson degrees of freedom that are necessary to render the Walecka model (power-counting) renormalizable.

III. MEAN-FIELD APPROXIMATION

Now we perform an alternative procedure to derive the NLPC model from the meson-exchange MNLW one. Here we perform the largely used mean-field approach (MFA), instead of the infinite meson mass limit taken in the previous section. We also use the no-sea approximation, i.e., we consider only the valence Fermi states. We will show that the EOS of the MNLW model are exactly the same of the NLPC one.

For infinite nuclear matter the energy density, and pressure of the NLPC models are given respectively by

$$\mathcal{E} = \frac{1}{2}{G'_V}^2\rho^2 + \frac{1}{2}{G'_s}^2\rho_s^2 + \frac{2}{3}A'\rho_s^3 + \frac{3}{4}B'\rho_s^4 + \frac{\gamma}{2\pi^2}\int_0^{k_F} k^2(k^2+M^{*2})^{1/2}dk \tag{52}$$

and

$$P = \frac{1}{2}{G'_V}^2\rho^2 - \frac{1}{2}{G'_s}^2\rho_s^2 - \frac{2}{3}A'\rho_s^3 - \frac{3}{4}B'\rho_s^4 + \frac{\gamma}{6\pi^2}\int_0^{k_F} \frac{k^4}{(k^2+M^{*2})^{1/2}}, \tag{53}$$

with the vector, and the scalar density defined as

$$\rho = \frac{\gamma}{2\pi^2}\int_0^{k_F} k^2 dk \quad \text{and} \tag{54}$$

$$\rho_s = \frac{\gamma}{2\pi^2}\int_0^{k_F} \frac{M^*}{(k^2+M^{*2})^{1/2}}k^2 dk, \tag{55}$$

with k_F being the Fermi momentum, $\gamma = 4$ for symmetric nuclear matter and $\gamma = 2$ for neutron matter. The nucleon effective mass reads

$$M^* \equiv M - {G'_s}^2\rho_s - A'\rho_s^2 - B'\rho_s^3. \tag{56}$$

Let us now start with the derivation of the MNLW equations of state first rewriting its Lagrangian, Eq. (30), as follows

$$\begin{aligned}\mathcal{L}_{MNLW} &= \bar\psi\left(i\gamma^\mu\partial_\mu - M\right)\psi + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu - g_V\bar\psi\gamma^\mu V_\mu\psi \\ &- \frac{1}{2}\left(m_s\phi + \frac{g_s}{m_s}\bar\psi\psi\right)^2 + \frac{g_s^2}{2m_s^2}\left(\bar\psi\psi\right)^2 - \frac{A'}{3}\left(\frac{m_s}{g_s}\phi + \bar\psi\psi\right)^3 + \frac{A'}{3}\left(\bar\psi\psi\right)^3 \\ &- \frac{B'}{4}\left(\frac{m_s^2}{g_s}\phi + \bar\psi\psi\right)^4 + \frac{B'}{4}\left(\bar\psi\psi\right)^4 .\end{aligned} \tag{57}$$

By knowing that a field translation does not alter the physical content of the model, we define

$$\lambda \equiv \frac{m_s^2}{g_s}\phi + \bar{\psi}\psi \,. \tag{58}$$

With this definition, the MNLW Lagrangian acquires the following form,

$$\begin{aligned}
\mathcal{L}_{MNLW} &= \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu - g_V\bar{\psi}\gamma^\mu V_\mu\psi - \frac{1}{2}{G_s'}^2\lambda^2 \\
&+ \frac{1}{2}{G_s'}^2\left(\bar{\psi}\psi\right)^2 - \frac{A'}{3}\lambda^3 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 - \frac{B'}{4}\lambda^4 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4 \\
&+ \frac{G_s'}{2m_s^2}\left(\partial^\mu\lambda - \partial^\mu\bar{\psi}\psi\right)\left(\partial_\mu\lambda - \partial_\mu\bar{\psi}\psi\right),
\end{aligned} \tag{59}$$

where $G_s' = g_s/m_s$.

In the MFA, the scalar and vector mesonic fields are replaced by their average values,

$$\lambda \to \langle\lambda\rangle \equiv \lambda \tag{60}$$

$$V^\mu \to \langle V^\mu\rangle \equiv \delta^{\mu 0}V^0 \,. \tag{61}$$

Still in this approximation, we use the $\bar{\psi}\psi$ ground-state expectation value. It is also assumed that the system is spatially uniform, so that the derivative terms of λ and $\bar{\psi}\psi$ disappear. Therefore, Eq. (59) becomes,

$$\begin{aligned}
\mathcal{L}_{MNLW}^{(MFA)} &= \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi + \frac{1}{2}m_V^2 V_0^2 - g_V\bar{\psi}\gamma^0 V_0\psi - \frac{1}{2}{G_s'}^2\lambda^2 + \frac{1}{2}{G_s'}^2\left(\bar{\psi}\psi\right)^2 \\
&- \frac{A'}{3}\lambda^3 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 - \frac{B'}{4}\lambda^4 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4 \,.
\end{aligned} \tag{62}$$

The independent fields of this theory may be taken as λ, $\bar{\psi}$, ψ and V^0. From the Euler-Lagrange equations one obtains the equations of motion for the fields,

$$\lambda({G_s'}^2 + A'\lambda + B'\lambda^2) = 0 \,, \tag{63}$$

$$V_0 = \frac{g_V}{m_V^2}\bar{\psi}\gamma_0\psi \tag{64}$$

and

$$[i\gamma^\mu\partial_\mu - g_V\gamma^0 V_0 - (M - {G_s'}^2(\bar{\psi}\psi) - A'(\bar{\psi}\psi)^2 - B'(\bar{\psi}\psi)^3)]\psi = 0 \,. \tag{65}$$

Now, substituting $\bar{\psi}\psi$ and $\bar{\psi}\gamma_\mu\psi$ by their respective mean values, we have

$$V_0 = \frac{g_V}{m_V^2}\left\langle\bar{\psi}\gamma_0\psi\right\rangle = \frac{g_V}{m_V^2}\rho \tag{66}$$

and consequently, Eq. (65) may be rewritten as

$$[i\gamma^\mu\partial_\mu - \gamma^0 {G'_V}^2\rho - (M - {G'_s}^2\rho_s - A'\rho_s^2 - B'\rho_s^3)]\psi = 0, \tag{67}$$

where $G'^2_V = g_V^2/m_V^2$, $G'^2_s = g_s^2/m_s^2$, and $\rho_s = \langle\bar\psi\psi\rangle$. The above Dirac equation suggests the effective nucleon mass definition,

$$M^* = M - {G'_s}^2\rho_s - A'\rho_s^2 - B'\rho_s^3, \tag{68}$$

that is exactly the same expression given in Eq. (56).

The mean-field equations of state will come from the energy-momentum tensor,

$$\begin{aligned} T_{\mu\nu}^{(MFA)} &= -g_{\mu\nu}\Big[\bar\psi(i\gamma^\alpha\partial_\alpha - g_V\gamma^0 V_0 - M)\psi + \frac{1}{2}m_V^2 V_0^2 - \frac{1}{2}{G'_s}^2\lambda^2 + \frac{1}{2}{G'_s}^2(\bar\psi\psi)^2 \\ &- \frac{A'}{3}\lambda^3 + \frac{A'}{3}(\bar\psi\psi)^3 - \frac{B'}{4}\lambda^4 + \frac{B'}{4}\left(\bar\psi\psi\right)^4\Big] + i\bar\psi\gamma_\mu\partial_\nu\psi \\ &= -g_{\mu\nu}\Big[-\frac{1}{2}{G'_s}^2(\bar\psi\psi)^2 - \frac{2A'}{3}(\bar\psi\psi)^3 - \frac{3B'}{4}(\bar\psi\psi)^4 + \frac{1}{2}m_V^2V_0^2 - \frac{1}{2}{G'_s}^2\lambda^2 \\ &- \frac{A'}{3}\lambda^3 - \frac{B'}{4}\lambda^4\Big] + i\bar\psi\gamma_\mu\partial_\nu\psi\,. \end{aligned} \tag{69}$$

The density energy is obtained from

$$\begin{aligned} \mathcal{E} &= \left\langle T_{00}^{(MFA)}\right\rangle \\ &= \frac{1}{2}{G'_s}^2\rho_s^2 + \frac{2A'}{3}\rho_s^3 + \frac{3B'}{4}\rho_s^4 - \frac{1}{2}{G'_V}^2\rho^2 + \frac{1}{2}{G'_s}^2\lambda^2 + \frac{A'}{3}\lambda^3 + \frac{B'}{4}\lambda^4 + i\left\langle\bar\psi\gamma_0\partial_0\psi\right\rangle, \end{aligned} \tag{70}$$

where we have used Eq. (66).

The quantity $i\left\langle\bar\psi\gamma_0\partial_0\psi\right\rangle$ is found by using the dispersion relation $k_0 = g_V V_0 + (k^2 + M^{*2})^{1/2} = {G'_V}^2\rho + (k^2 + M^{*2})^{1/2}$, where k_0 is the fourth energy-momentum component. This leads to

$$\begin{aligned} \mathcal{E} &= \frac{1}{2}{G'_V}^2\rho^2 + \frac{1}{2}{G'_s}^2\rho_s^2 + \frac{2A'}{3}\rho_s^3 + \frac{3B'}{4}\rho_s^4 + \frac{1}{2}{G'_s}^2\lambda^2 + \frac{A'}{3}\lambda^3 + \frac{B'}{4}\lambda^4 \\ &+ \frac{\gamma}{2\pi^2}\int_0^{k_F}(k^2 + M^{*2})^{1/2}k^2 dk\ . \end{aligned} \tag{71}$$

The pressure is obtained by

$$\begin{aligned} P &= \frac{1}{3}\left\langle T_{ii}\right\rangle \\ &= \frac{1}{2}{G'_V}^2\rho^2 - \frac{1}{2}{G'_s}^2\rho_s^2 - \frac{2A'}{3}\rho_s^3 - \frac{3B'}{4}\rho_s^4 - \frac{1}{2}{G'_s}^2\lambda^2 - \frac{A'}{3}\lambda^3 - \frac{B'}{4}\lambda^4 + \frac{1}{3}i\left\langle\bar\psi\gamma_i\partial_i\psi\right\rangle. \end{aligned} \tag{72}$$

By extracting $i\left\langle\bar{\psi}\gamma_i\partial_i\psi\right\rangle$ from the the Dirac equation, the pressure can be written as follows

$$P = \frac{1}{2}{G'_V}^2\rho^2 - \frac{1}{2}{G'_s}^2\rho_s^2 - \frac{2A'}{3}\rho_s^3 - \frac{3B'}{4}\rho_s^4 - \frac{1}{2}{G'_s}^2\lambda^2 - \frac{A'}{3}\lambda^3 - \frac{B'}{4}\lambda^4 + \frac{\gamma}{6\pi^2}\int_0^{k_F}\frac{k^4}{(k^2+M^{*2})^{1/2}}dk\ . \tag{73}$$

The auxiliary λ field are decoupled from the fermionic sector. Its contribution either to the pressure or to the energy shall indeed be dropped by the physical requirement that the pressure goes to zero when k_F vanishes, implying that only the trivial solution, $\lambda = 0$, of Eq. (63) should be kept. Therefore the energy density and the pressure become, respectively

$$\mathcal{E} = \frac{1}{2}{G'_V}^2\rho^2 + \frac{1}{2}{G'_s}^2\rho_s^2 + \frac{2}{3}A'\rho_s^3 + \frac{3}{4}B'\rho_s^4 + \frac{\gamma}{2\pi^2}\int_0^{k_F}(k^2+M^{*2})^{1/2}k^2dk \tag{74}$$

and

$$P = \frac{1}{2}{G'_V}^2\rho^2 - \frac{1}{2}{G'_s}^2\rho_s^2 - \frac{2}{3}A'\rho_s^3 - \frac{3}{4}B'\rho_s^4 + \frac{\gamma}{6\pi^2}\int_0^{k_F}\frac{k^4}{(k^2+M^{*2})^{1/2}}dk. \tag{75}$$

Notice that the equations above of the MNLW model are identical to Eqs. (52), and (53) of the NLPC ones, what shows that the former can be also obtained from the mean-field approximation at the level of the EOS instead of the Lagrangian density framework shown by the hypermassive limit taken in the Section II.

IV. CONCLUSION

If one performs a hypermassive meson limit to usual NLW models, baryonic NLPC models are not obtained. We have shown that in order to obtain NLPC models, a modification of NLW models are needed, already at the level of the Lagrangian density. In this work we derived the point-coupling models from a modified NLW by using the hypermassive meson limit in the functional integral method. From this approach we shown how the linear PC models can be obtained from the Walecka ones, and in the same way, how the MNLW model generates the NLPC one. This relation among MNLW and NLPC is described as equivalences of the physical content of these models encoded in their irreducible algebra of fields in the infinite meson masses limit. In addition, we also use the no-sea approximation to construct the equations of state of the MNLW model. We remark that even in this alternative way (without taking the hypermassive meson limit) the EOS are exactly the same of the

NLPC ones. Therefore, to treat MNLW model either from the meson hypermassive limit or alternatively to treat directly from the mean field approach leads to the same NLPC models.

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